كلية الهندسة بشبرا
FACULTY OF ENGINEERING AT SHOUBRA

Geodesy 1B
Lecture 3
Coordinate Reference Systems

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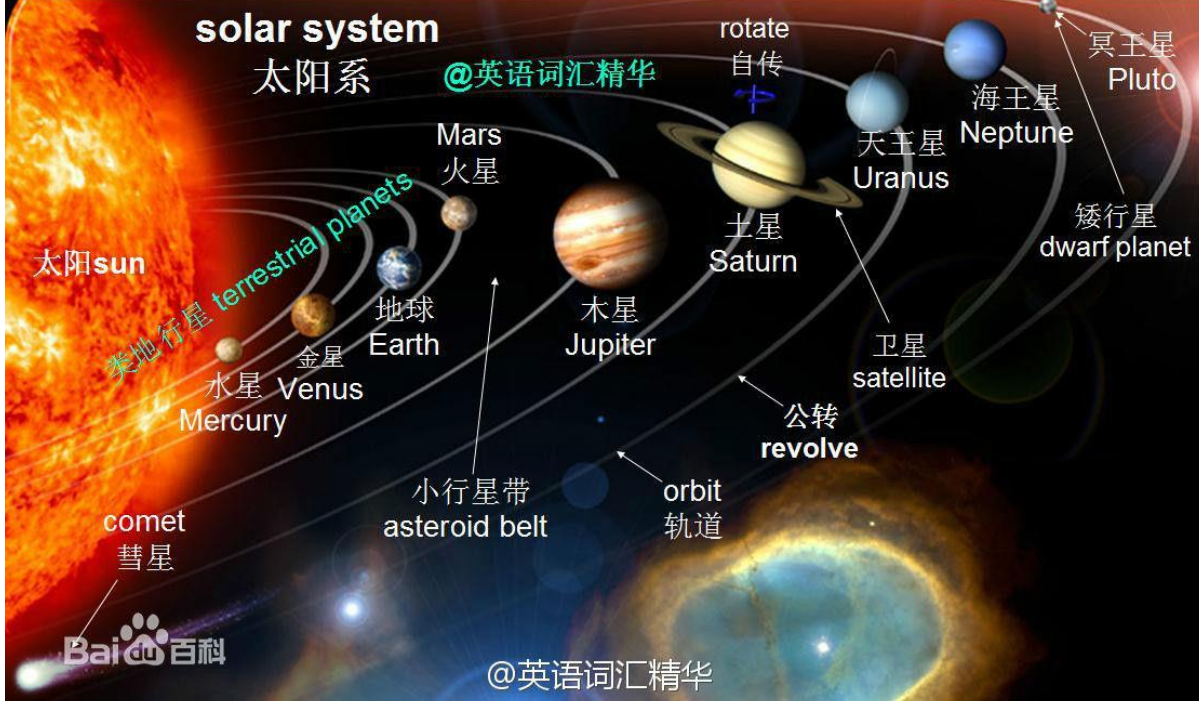
3.1 The Earth and its motions



(a)It moves with our galaxy in respect to other galaxies.

(b)It circulates with the solar system within our galaxy.

(c)It revolves around the sun, together with other planets.

(d)It rotates (spins) around its instantaneous axis of rotation.

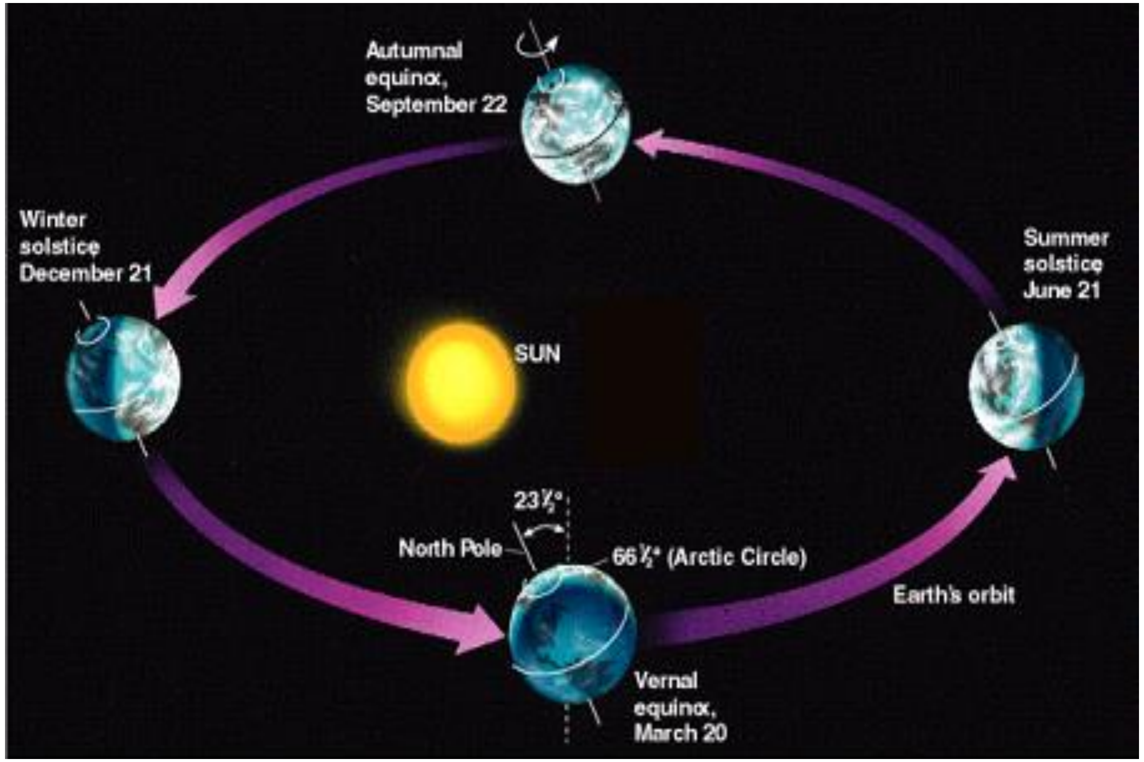
3.1.1 Earth's Annual Motion

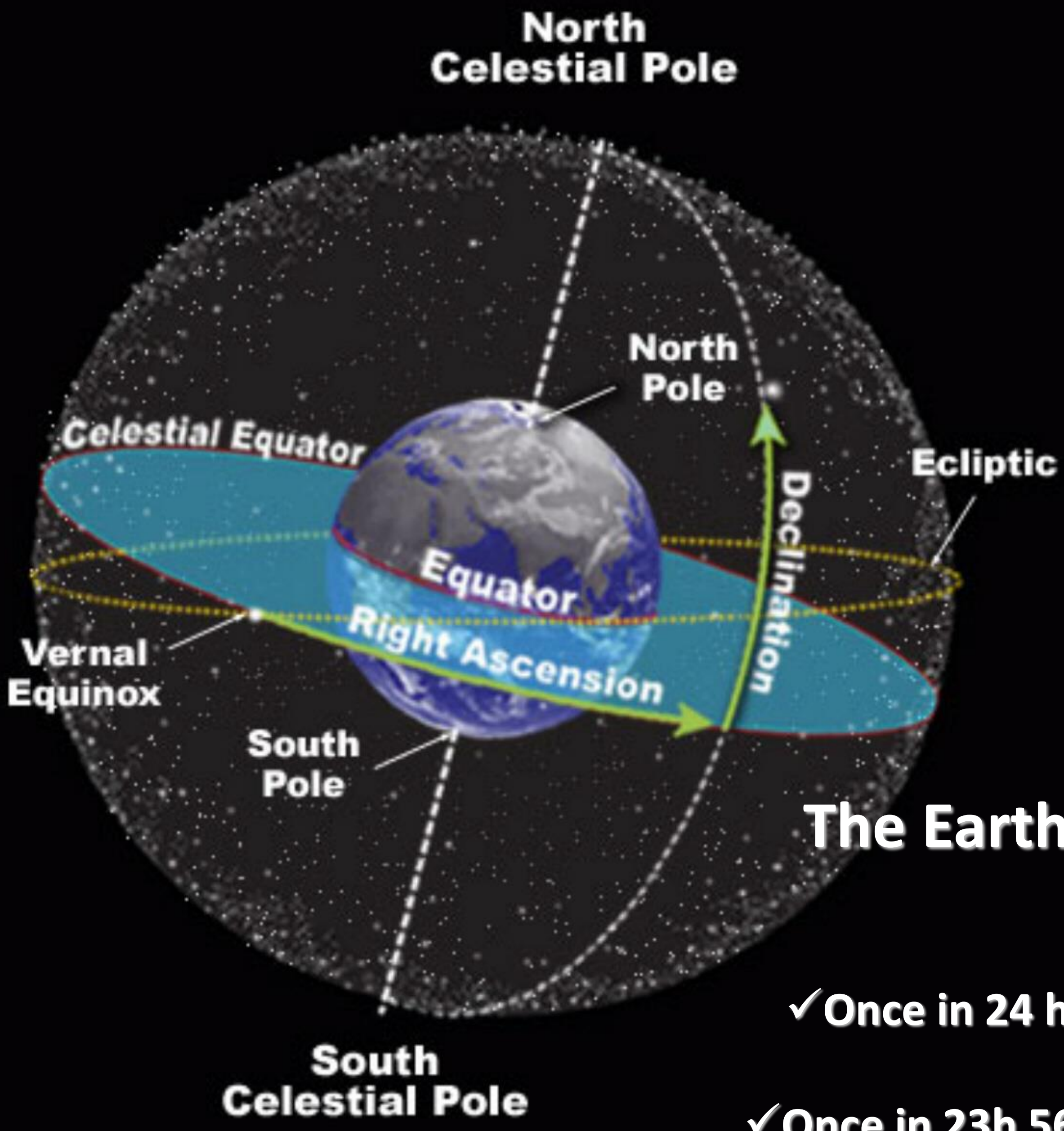
(a) The orbit is an ellipse

(b) A planet moves along its orbit with a constant area velocity.

(c) $a^3/T^2 = C$

Kepler's laws





The Earth spins about its axis:

✓Once in 24 hours w.r.t. the sun (solar day)

✓Once in 23h 56m w.r.t. the stars (sidereal day)

3.2 Coordinate Reference Systems

Brief intro

- It is known that by means of certain mathematical operations we can transfer our physical observations into geodetic information of position, azimuth, elevation, distance, or size and shape of the earth.
- A coordinate system is necessary for all our calculations , whether as an intermediate step or end result.

3.2.1. Natural Coordinate System $\underline{\Phi, \Lambda, H}$

- Astronomical observations for latitude, longitude, and azimuths are measured with reference to the direction of gravity at the point of observation.
- In the natural coordinate system the position of any point on the earth's surface can be fixed by observing its astronomic latitude, longitude, and its orthometric height

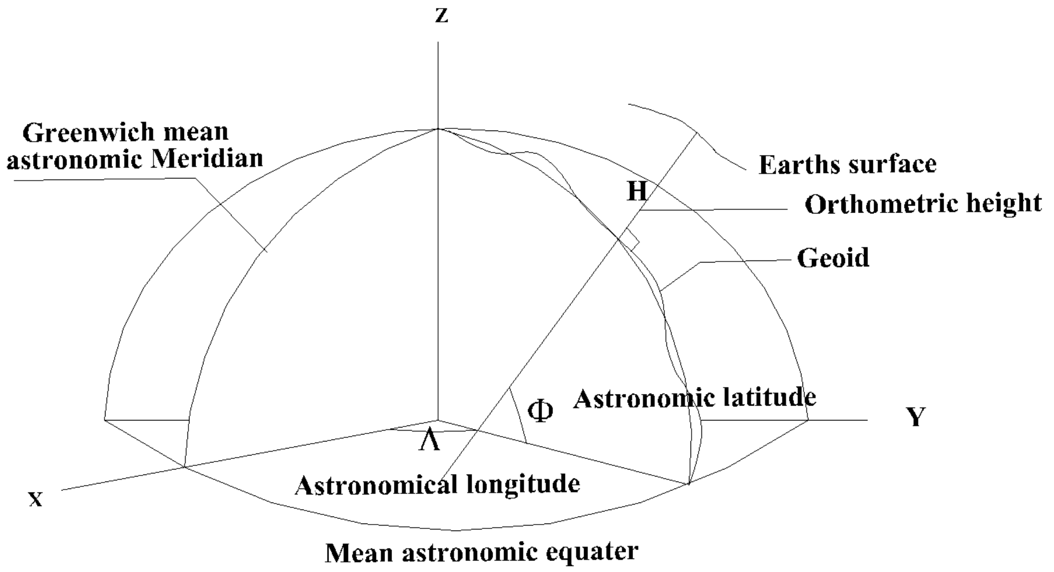


FIG 5-1

3.2.1. Natural Coordinate System

1) Astronomical Latitude Φ: is the angle between the equatorial plane and the direction of the vertical at the point of observation.

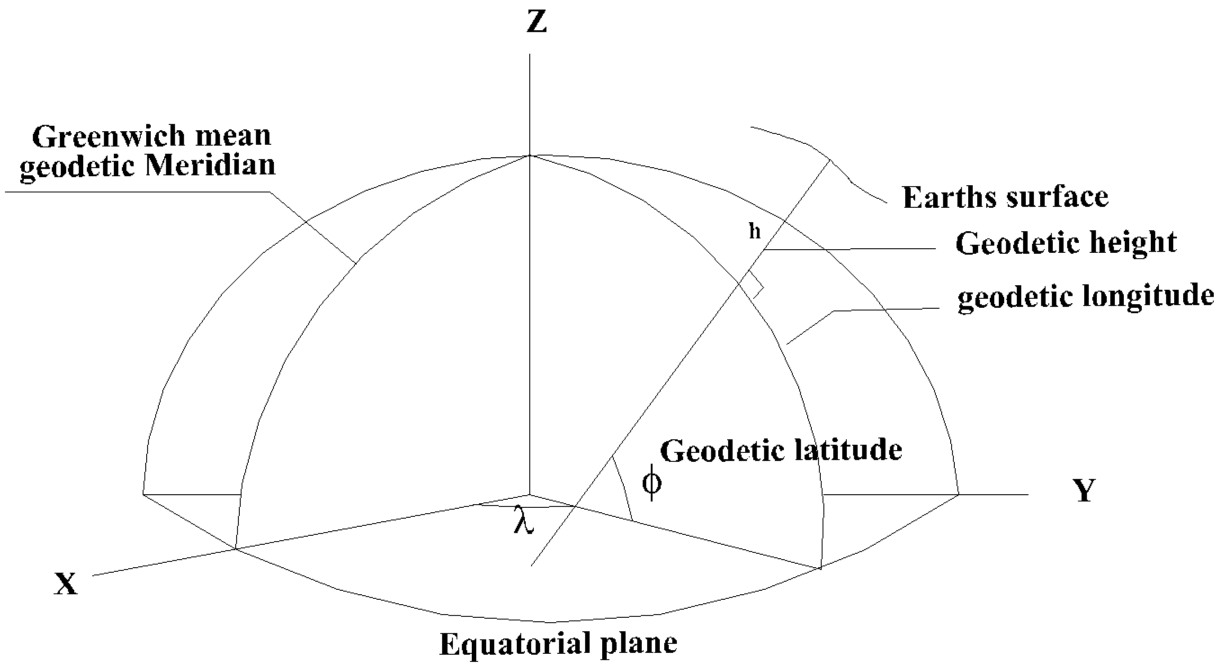
2) Astronomical Longitude Λ: is the angle between the meridian plane of the observation point and the meridian plane of Greenwich.

3) Orthometric Height H : is the height of a point above mean sea level. It is measured along the curved plumb line and obtained from spirit levelling and gravity observations.

3.2.2. Geodetic Coordination System ϕ, λ, h

- Since the deviations of the geoid from the reference ellipsoid are small and can be computed, it is convenient to add small reductions to the observed coordinate so that, values refer to an ellipsoid can be established, which are called geodetic coordinates.
- Geodetic

(Curvilinear, Geographic)



3.2.2. Geodetic Coordination System

1) Geodetic Latitude ϕ: is the angle between the ellipsoidal normal of the observers projected position on the geoid and the perpendicular to the mean rotation axis of the earth.

2) Geodetic Longitude λ: is the angle between the same ellipsoidal normal and Greenwich meridian plane.

3) Geodetic Height h: is the height of the observer above the reference ellipsoid, measured along the ellipsoidal normal.

The geodetic coordinates are determined from *Triangulation* or *Trilateration* observed on the earth surface, reduced to the ellipsoid.

They could also be obtained directly from the astronomic coordinates reduced to the used reference ellipsoid.

3.2.3.Rectangular (Cartesian) Coordinate System **X, Y, Z.**

- Generally, it is convenient to take the X-axis parallel to the meridian of Greenwich; the Y-axis has the longitude of 90° east of Greenwich, and the Z-axis parallel to the CIO (conventional international origin of polar motion). Ideally the origin of the rectangular coordinates system should be at the earth's center of gravity; the system is known as "Average Terrestrial Coordinate System". When the origin is at the geometric center of the ellipsoid, and not in the (C.G.) of the earth, it is known as "Geodetic Coordinate System"

3.2.3. Rectangular (Cartesian) Coordinate System

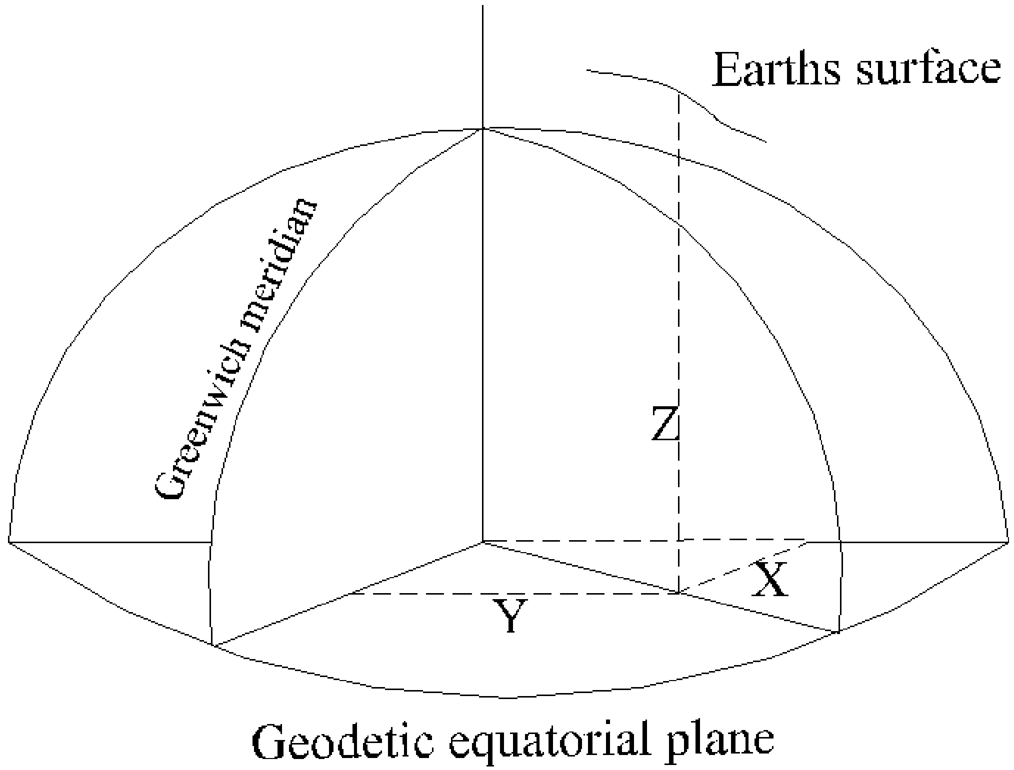


FIG 5-3

3.2.4. Local Coordinate System (Horizon System) U, V, W

- In this system the coordinates U, V, W are expressed as functions of the observed azimuth A, zenith distances Z & spatial distance S.

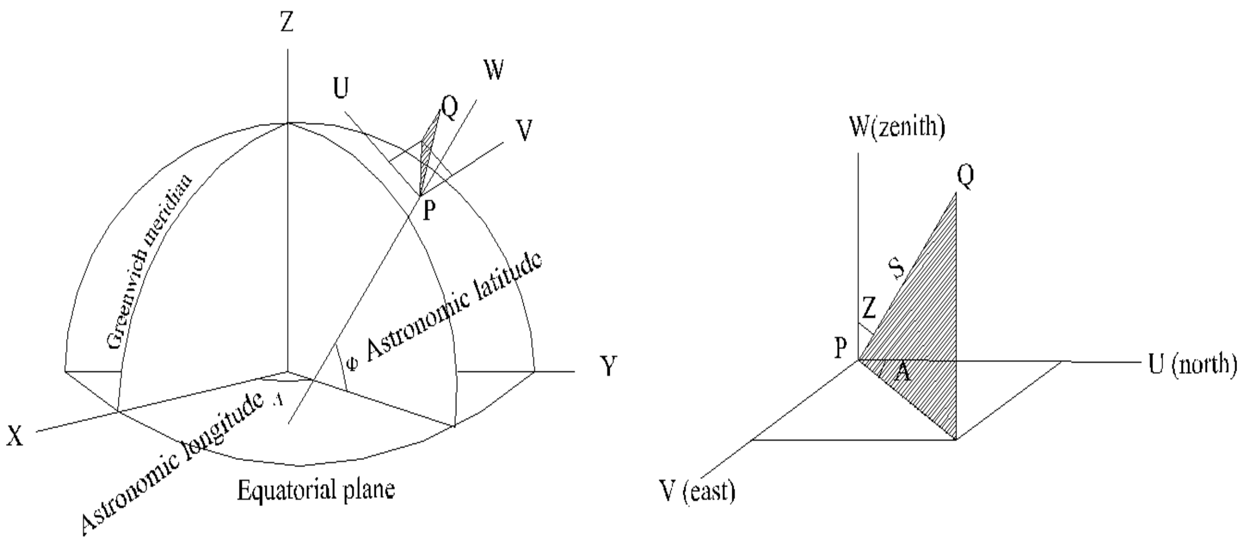


FIG 5-4

3.2.4. Local Coordinate System (Horizon System) U, V, W

illustrates the quantities of this system. The origin is considered to be at the observation station P. the positive U-axis points N-ward, the positive V-axis points eastward and the positive W-axis coincides with the outward direction of the plumb line.

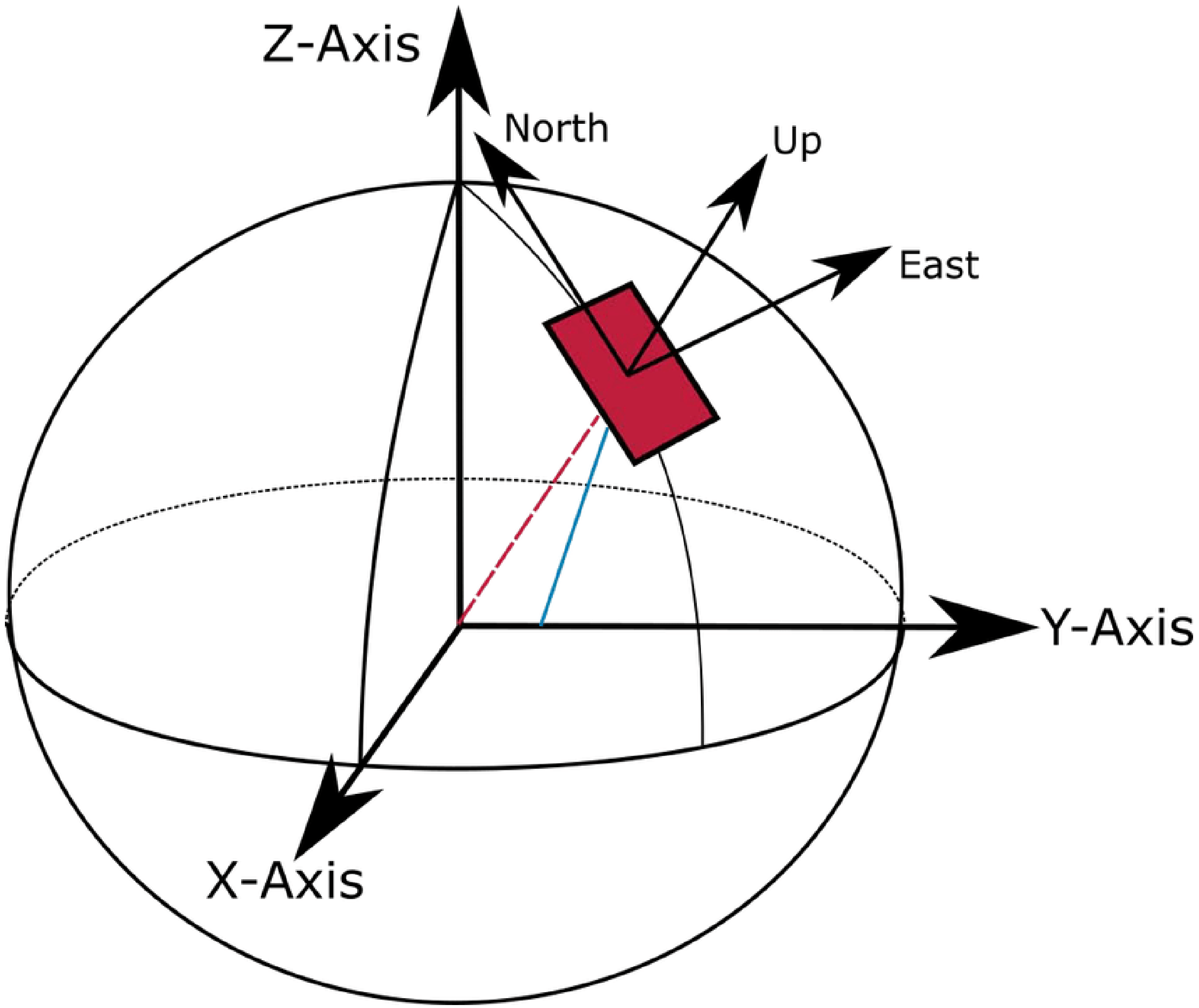
The coordinate equations of an object Q sighted in this system may be written simply by referring to the fig.

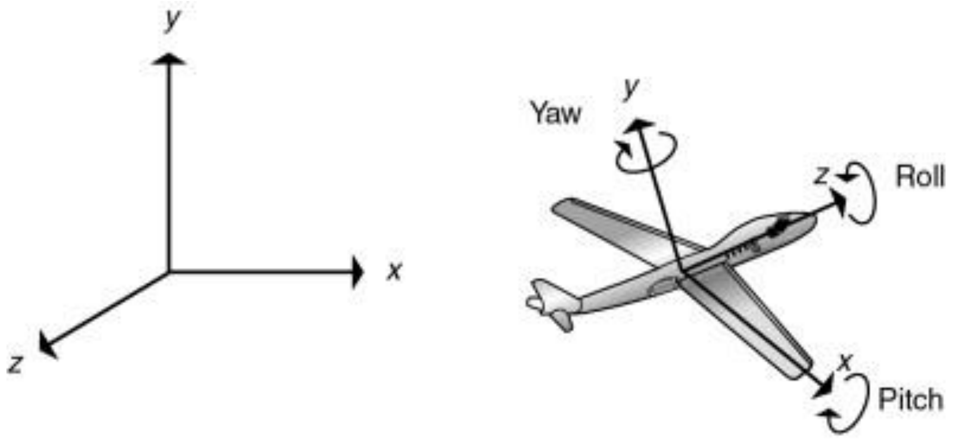
$$U = S \sin Z \cos A$$

$$V = S \sin Z \sin A \qquad (5\text{-}1)$$

$$W = S \cos Z$$

3.2.4. Local Coordinate System (forms)





Global coordinate system

Local coordinate system attached to object

3.3. Relations between different Reference Systems

3.3.1. Relation between Astronomic and Geodetic Coordinates

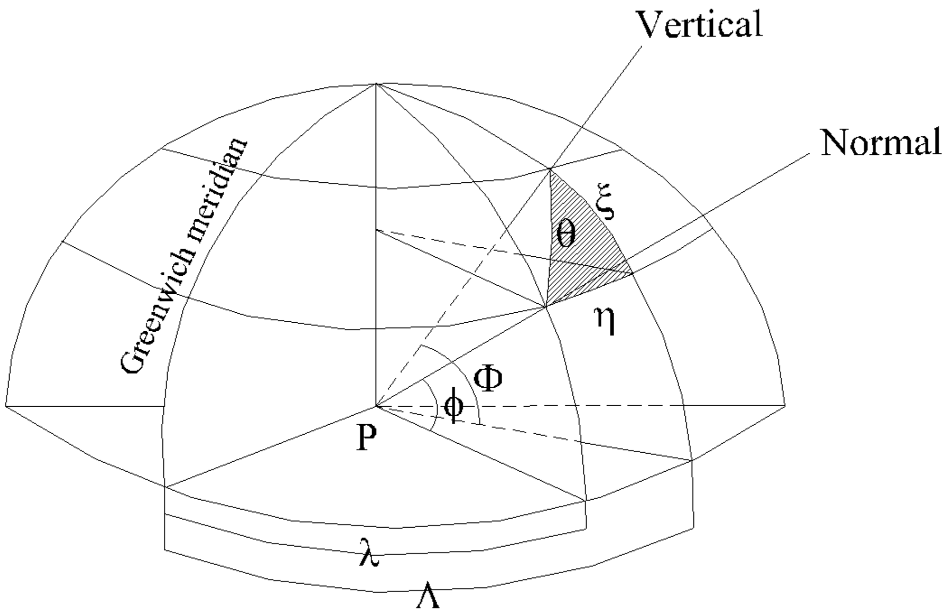


FIG 5-5

3.3.1. Relation between Astronomic and Geodetic Coordinates

Since the astronomical system depends on the direction of the vertical "actual gravity field", while the geodetic system depends on the direction of the ellipsoidal normal "normal gravity field", then the relation between both systems depends mainly on the difference between the two directions. The total difference between the two directions is the well-known deflection of the vertical θ. It has two components, a north-south component ξ and an east-west component η. We can read from figure (5-5) the following:

$$\xi = \Phi - \phi$$

$$\eta = (\Lambda - \lambda)\cos\phi$$

$$\theta = (\xi^2 + \eta^2)^{0.5}$$

$$h = H + N$$

3.3.2. Relation between Rectangular and Curvilinear Geodetic Coordinates:

$$X = (N + h)\cos\phi\cos\lambda$$

$$Y = (N + h)\cos\phi\sin\lambda$$

$$Z = (N(1 - e^2) + h)\sin\phi$$

3.3.2. Relation between Rectangular and Curvilinear Geodetic Coordinates(Inverse Procedure)

The computation of ϕ, λ, h from given X, Y, Z is more complicated because the three equations have four unknowns, N including ϕ. Accordingly, the computation could be done iteratively in addition to the direct solution. Many solutions, through iteration, were given for this problem, for example; *HIRVONEN & MORITZ 1963 BARTELME & MEISEL 1975, RAPP* and *KRAUSS 1976*. Also a non-iterative solution was given by *SUENKEL 1976.*

3.3.2. Relation between Rectangular and Curvilinear Geodetic Coordinates(Inverse Procedure)

$$R_P = (X^2 + Y^2)^{0.5} = (N + h)\cos\phi$$

hence

$$h = \frac{R_P}{\cos\phi} - N$$

Equation (5-5) may be transformed into

$$Z = (N - \frac{a^2 - b^2}{a^2} N + h)\sin\phi$$

$$Z = (N + h - e^2 N)\sin\phi$$

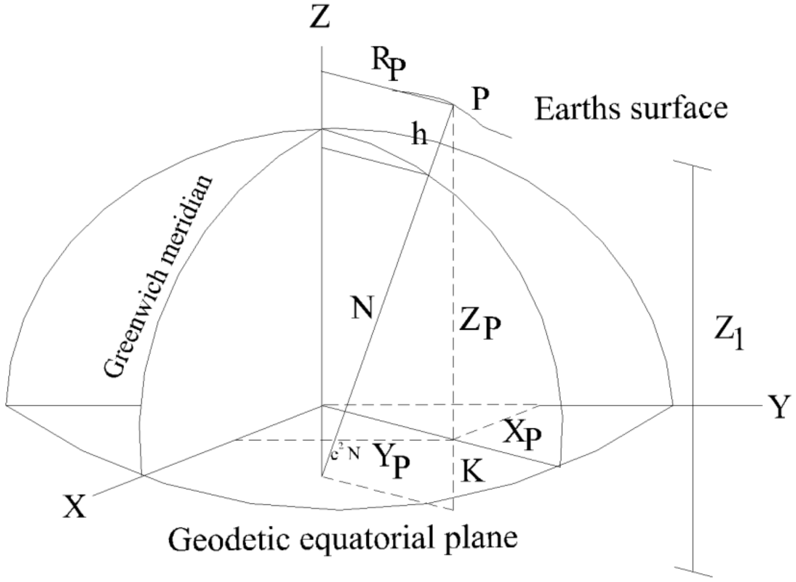


FIG 5-7

3.3.2. Relation between Rectangular and Curvilinear Geodetic Coordinates(Inverse Procedure)

where

$$e^2 = {(a^2 - b^2)}\Big/{a^2}$$

Dividing this equation by the above expression for R_P we get

$$\frac{Z}{R_P} = (1 - e^2 \frac{N}{N+h}) \tan\phi$$

so that

$$\tan\phi = \frac{Z}{R_P}(1 - e^2 \frac{N}{N+h})^{-1} \qquad (5\text{-}7)$$

Given X, Y, Z and hence, R_P equations (5-6) and (5-7) may be solved iteratively for h and ϕ.

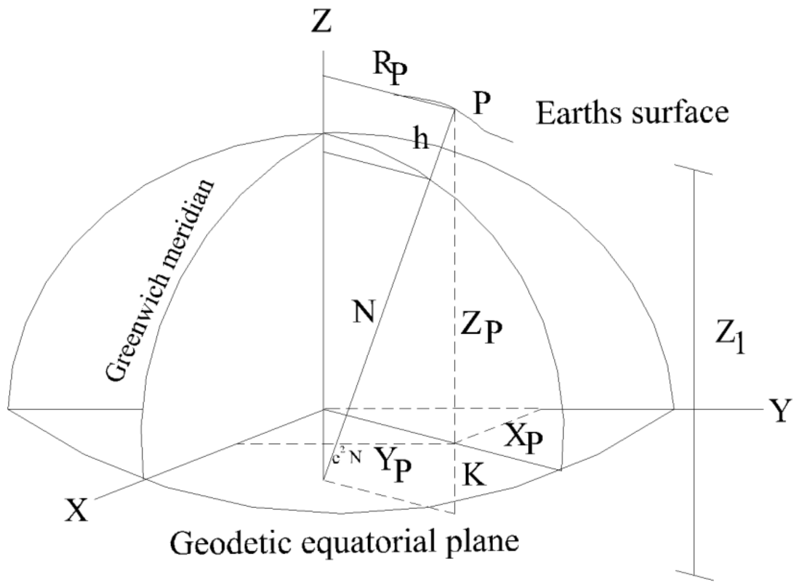


FIG 5-7

3.3.2. Relation between Rectangular and Curvilinear Geodetic Coordinates(Inverse Procedure)

As a first approximation, we set $h = 0$ in (5-7), obtaining

Using ϕ_1, we compute an approximate value N_1 by means of

$$N_1 = \frac{a}{(1 - e^2 \sin^2 \phi_1)^{0.5}} \tag{5-8}$$

and introduce this value of N_1 in equation (5-6) to get an approximate value h_1.

$$h_1 = \frac{R_P}{\cos \phi_1} - N_1$$

Now, as a second approximation, we set $h = h_1$ in (5-7) obtaining

$$\tan \phi_2 = \frac{Z}{R_P}(1 - e^2 \frac{N_1}{N_1 + h_1})^{-1}$$

3.3.2. Relation between Rectangular and Curvilinear Geodetic Coordinates(Inverse Procedure)

z

Using ϕ_2, improved values for N & h are found, etc. This procedure is repeated until the values of ϕ & h remain practically constant. The third value λ can be easily calculated from

$$\tan \lambda = Y/X$$

(5-9)

$$\tan \phi_1 = \frac{Z}{R_P}(1-e^2)^{-1}$$

3.3.3. Relation between Horizon and Rectangular System

Then, the final form are achieved by combining these relations together as follows

$$\begin{vmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{vmatrix} = \begin{vmatrix}-\sin\Phi\cos\Lambda & -\sin\Lambda & \cos\Phi\cos\Lambda\\ -\sin\Phi\sin\Lambda & \cos\Lambda & \cos\Phi\sin\Lambda\\ \cos\Phi & 0 & \sin\Phi\end{vmatrix}\cdot\begin{vmatrix}u\\ v\\ w\end{vmatrix}$$

Or in matrix notation $\Rightarrow X = R^T \cdot u$

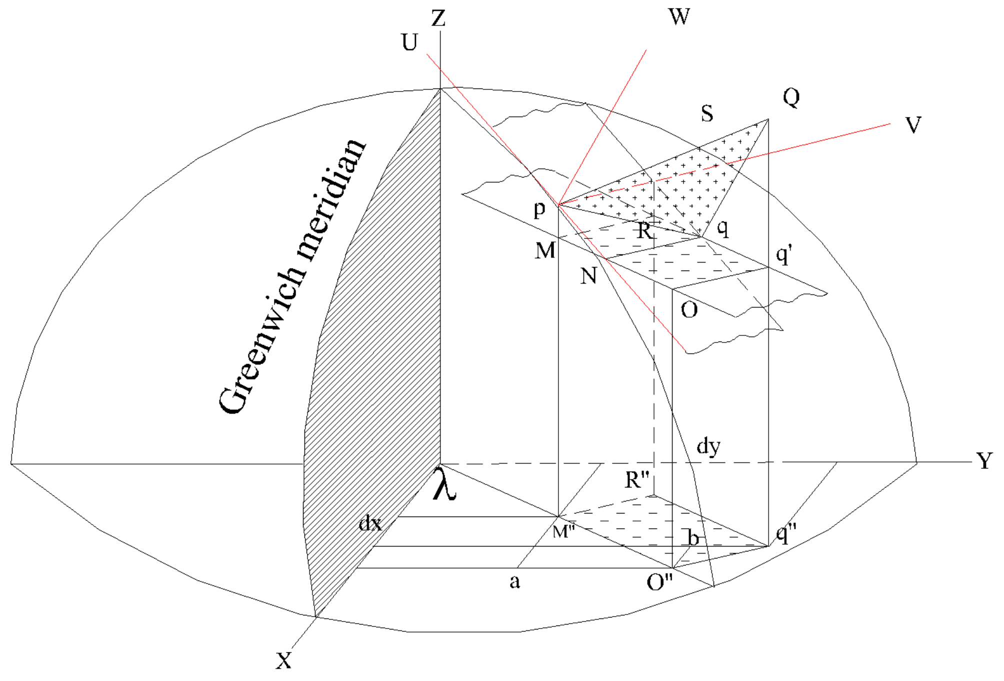


FIG 5-8

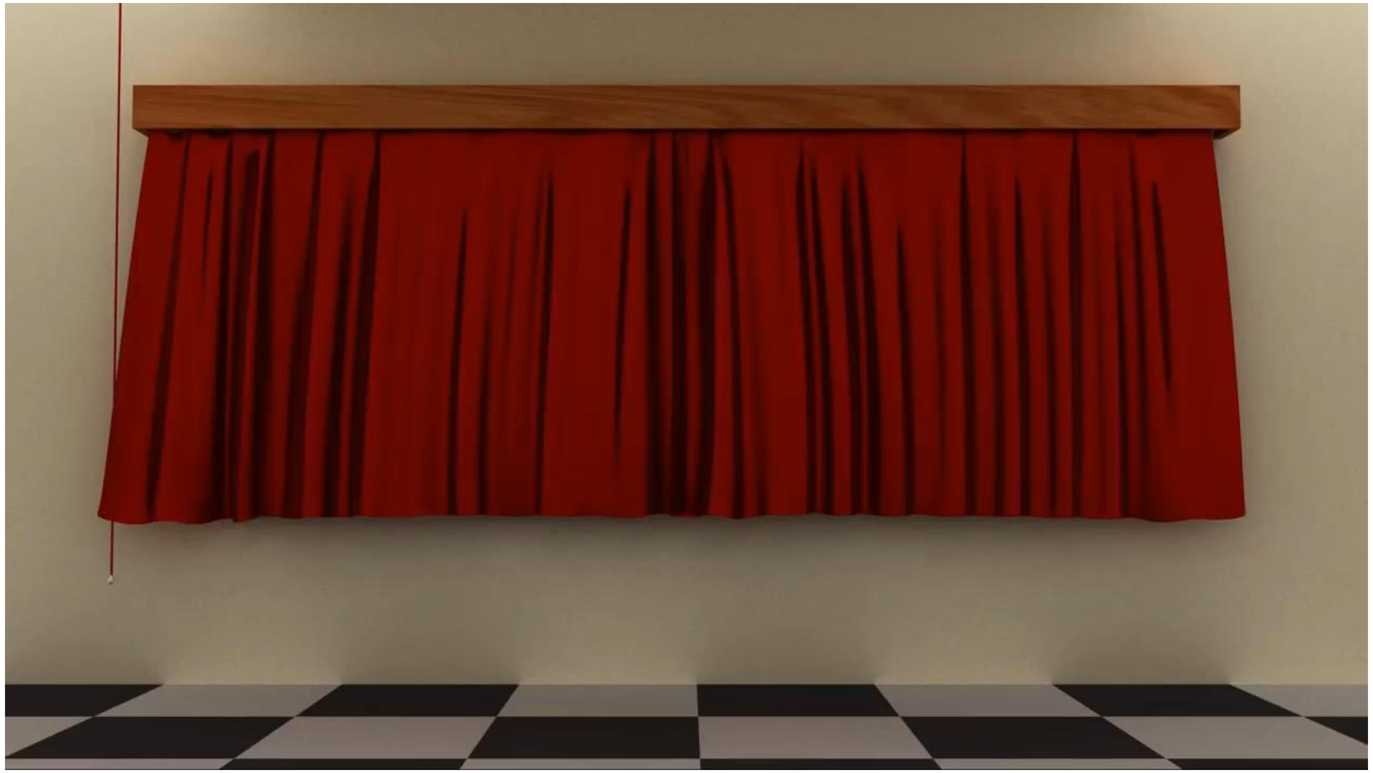
Review and Questions

(1) What kind of motions is the Earth experiencing?

(2) Describe Coordinate reference systems.

(3) Summarize Transformation between different coordinate systems

Video (Spherical Coordinate System)



https://www.youtube.com/watch?v=FDyenWWlPdU

Exercise one (Deadline March 7th, 2023)

To write a computer programme for the transformation between Cartisian coordinate (X,Y,Z) and geodetic coordinate (Lat,Long,H).

Testing data set (WGS84)

A001	-2235714.3384	4583893.6967	3817435.2874
A002	-2234659.4755	4582817.5148	3819312.6948
A003	-2236331.6313	4583720.8507	3817276.9676

A001	37.000000000	116.000000000	70.000
A002	37.011654728	115.594072975	58.508
A003	36.595364778	116.002550281	66.798